Interference

It is the process whereby two or more waves of the same frequency or wavelength combine to form a wave whose amplitude is the sum of the amplitudes of the interfering waves. The interfering waves can be electromagnetic, acoustic, or water waves.

Superposition of waves

Consider a region in space where two or more waves pass through at the same time.according to the super position principle,the net displacement is simply given by the vector or the algebraic sum of the individual displacements.interference is the combination of two or more waves to form a composite wave.

Constructive interference:occurs when two waves are in phase. To be in phase,the points on the wave must have$\Delta\emptyset=(2\pi)m$,where m is an integer.

For constructive interference:$\Delta l=m\lambda$ where m:is an integer

Destructive interference: occurs when two waves are a half cycle out of phase. To be out of phase the points on the wave must have :$\Delta\emptyset=(2\pi)(m+1/2)\lambda$

Conditions of interference

The following four conditions must be turn in order for an interference pattern to be observed.

1-the source must be coherent (has a constant phase relationship).

2-wavelengths must be the same(monochromatic).

3-the principle of superposition must apply.

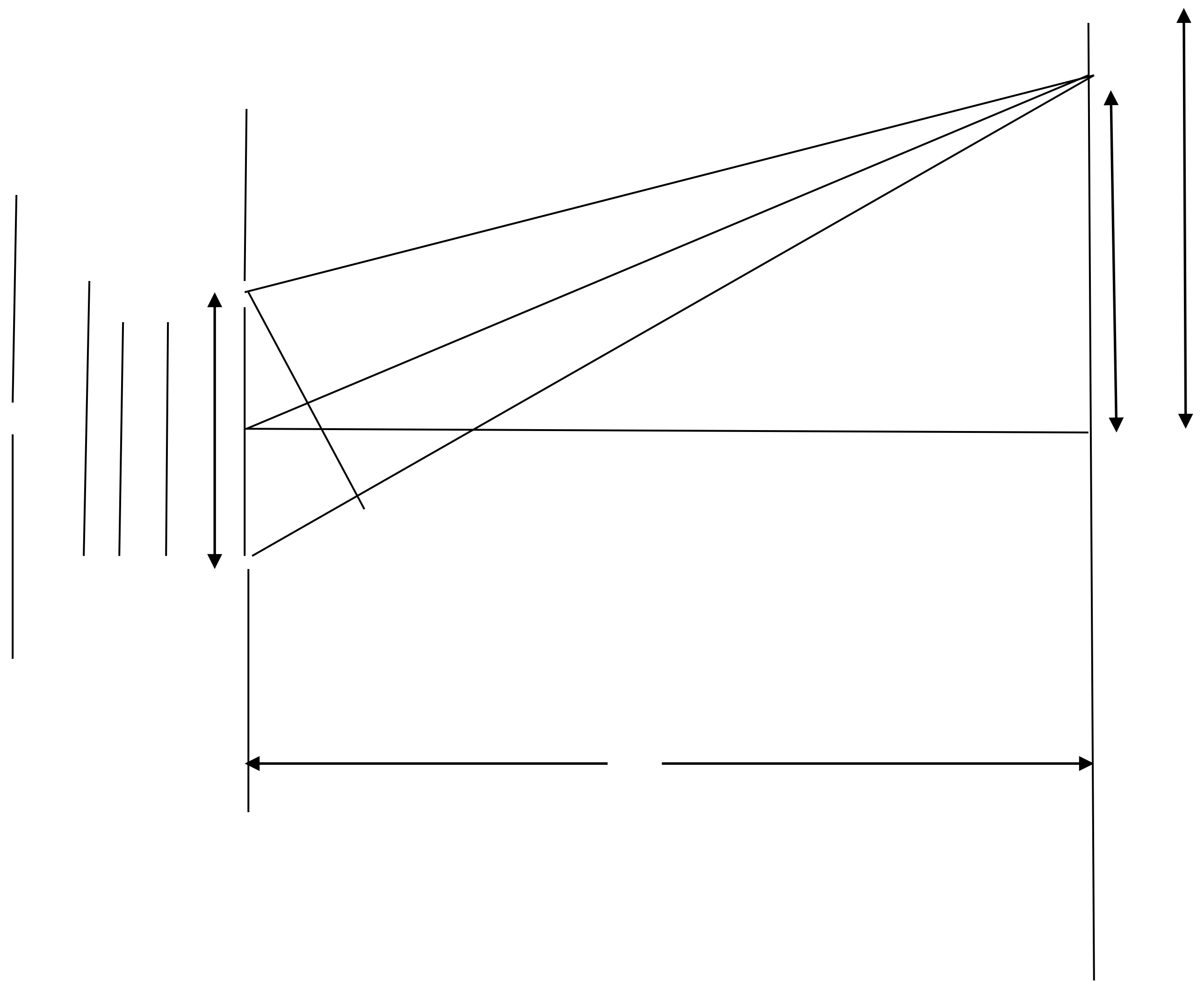
4-the wave have the same polarization state.

Huygen' s principle

Huygen 's proposed that:Awave front may be regarded as a new source of waves.

Young' s experiment

In young's experiment of interference the light is passes through a pinhole (s_0)in the sheet (A),the at a considerable distance away,through two pinhole (s_1)and (s_2)in sheet (B),the combine in the region after the sheet (B),the interference phenomenon will be happened between the two transmitted waves from two pinholes, this phenomenon can be observed as a fringe mode on the sheet(c).



Now ,suppose that the light is monochromatic with a wavelength ʎ,the distance d between two pinholes smaller than the distance D between the two sheets C,B.

The condition for maximum intensity in point (p) on sheet c is:

$d\sin\theta=m\lambda$,m=0,1,2,3………

and the condition for the minimum intensity is:

$d\sin\theta=(m+1/2)\lambda$,m=0,1,2,3………

Interference fringes from a double slit

The geometry of the double slit interference is shown in fig.(1)

Consider a light that falls on the screen at a point p which it is at a distance (x_n)from the point 0 that lies on the screen ,D is the perpendicular distance from the double slit system. The two slits are separated by a distance d.

Now we derive the fringe separation equation:-

At point 0,the path difference between the two waves:

$S_2 0 - S_1 0 = 0$ …………1)

At point p,the position of the nth order bright fringe (or maxima),the path difference between the two sources $S_1$and S_2 must differ by a whole number of wavelengths:

Path difference:$S_2p-S_1p=m\lambda$ …….2)

As a distance D is very much larger than d, the path difference (S_2p-S_1p) can be approximated by dropping a perpendicular line(S_1N) to S_2P such that S_1p –Np.

Path difference $S_2P-S_1p - S_2N=m\lambda$ --------3)

And from geometry:

$S_2N=d\sin\Theta$ ……….4)

Where d: is the distance between the centers of the two slits.

Equating eqns.(3,4) yields:

$d\sin\Theta=m\lambda$ …….5)

$\sin\Theta=m\lambda/ d$ ……6)

but fom geometry: $\tan\Theta = X_n / D$ X_n: is the distance of nth order fringe from the central axis.

Since Θ is usually very small , $\tan\Theta \sim \sin\Theta$

i.e $X_n / D = m\lambda / d$

or $X_n = m\lambda D / d$ …………7)

thus the separation between adjacent fringes is:

$$\Delta X = X_{n+1} - X_n = (m+1)\lambda D / d - m\lambda D / d$$

Fringe separation: $\boxed{\Delta X = \lambda D / d}$ for bright fringe

Intensity distribution in the fringe system

Note that the edges of the bright fringes are not sharp, there is a gradual change from bright to dark.so we have knew the locations of only the centers of the bright and dark fringes on a distant screen.

Let us now direct our attention to the intensity of the light at oher points between the positions of maximum constructive and destructive intereference. In ther words, we now calculate the distribution of light associated with the double-slit intereference pattern.

Suppose that the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ω anda constant phase difference $\varnothing$ .the total magnitude of the electric fields at point p on the screen is the superposition of the two waves.

Assuming that the two waves have the same amplitude E_0,we can write the magnitude of the electric field at point p due to each wave separately as:

$E_1=E_0\sin\omega t$, $E_2= E_0\sin(\omega t+\varnothing)$

Using the superposition principle,we can obtain the magnitude of the resultant electric field at point p:

$E_p=E_1+E_2= E_0[\sin(\omega t+\varnothing)+\sin\omega t$

From trigonometric identity:

$\sin A+\sin B=2\sin(A+B /2)\cos (A-B/2)$

$A=(\omega t+\varnothing)$,$B=\omega t$

$E_P=2E_0\sin(\omega t+\varnothing+\omega t /2)\cos (\omega t+\varnothing-\cancel{\omega t} /2)$

$E_P=2E_0\cos (\varnothing/2)\sin (\omega t+\varnothing/2)$

This indicates that the electric field at point p has the same frequency ω as the light at the slits. But that the amplitude of the field is multiplied by the factor $[2\cos(\varnothing/2)]$.

To check the consistency of this result note that:

If $\varnothing=0, 2\pi, 4\pi, \ldots\ldots\ldots$

$E_P=2E_0$

This corresponds to the condition for maximum constructive interference.

Likewise if $\varnothing=\pi, 3\pi, 5\pi, \ldots\ldots\ldots$, then the magnitude of the electric field at point p is zero.

This corresponds to destructive interference.

To obtain an expression for the light intensity at point p:

$I \propto E_P^2$

$$I=E_P^2=4E_0^2 \cos2(\varnothing/2)\sin^2(\omega t+\varnothing/2)$$

Most light-detecting instruments measure time-averaged light intensity,

The time average of $\sin^2(\omega t+\varnothing/2)$ over one cycle $=1/2$

$I=E_P^2=4E_0^2\,(1/2)\cos^2(\varnothing/2)$

$I=2E_0^2\cos^2(\varnothing/2)$

$I=I_0\cos^2(\varnothing/2)$ (*) where I_0:is the maximum intensity on the

Screen$=2E_0^2$

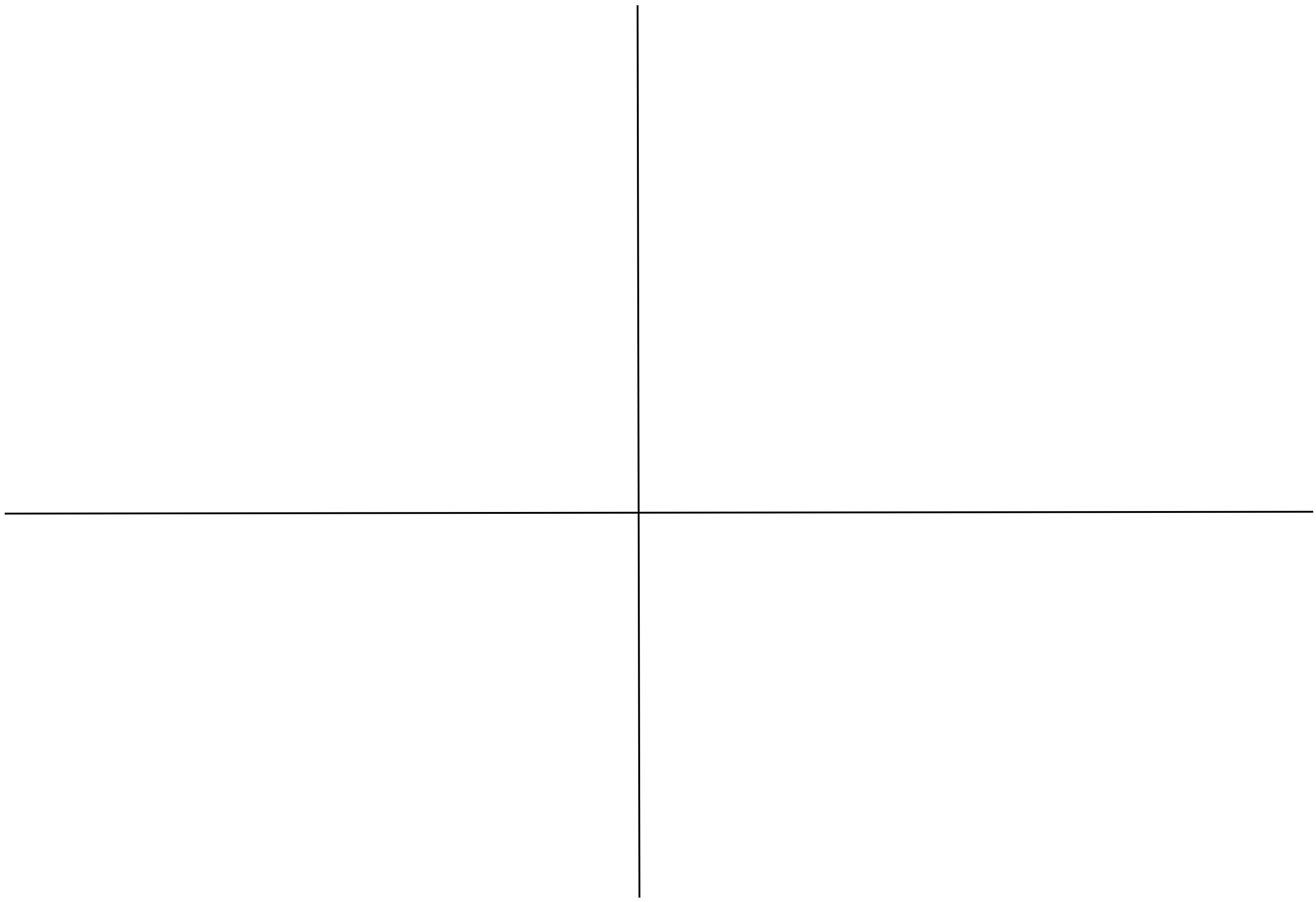
For constructive int.: path difference $\delta=\lambda$ which corresponds to phase shift of $\varnothing=2\pi$. This implies that the ratio of path difference to wavelength is equal to the ratio of phase shift to 2π.

$d'/\lambda = \varnothing/2\pi \Longrightarrow \varnothing = 2\pi d'/\lambda$

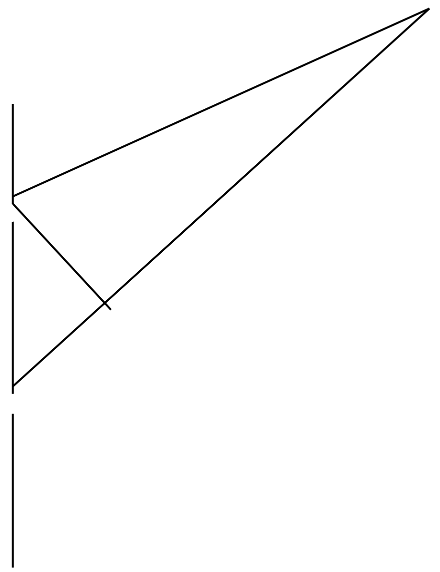
$$\boxed{\varnothing = 2\pi d \sin\Theta/\lambda}$$

From eq.(*) $\boxed{I/I_0 = \cos^2(\pi d \sin\Theta/\lambda)}$

The plot of the ratio I/I_0 as a function of $d\sin\Theta/\lambda$



Ex:In the figure below, there are two coherent sources emit light of a wavelength λ=0.1m,and the emitted waves interfered at point p in the same time. What is the type of the obtained interference at this point when one of the two waves travel an optical path (3.2m),and the other an optical path (3m).



Sol.

l_1=3m,l_2=3.2m

prob.1:**$\Delta l=(m+1/2)\lambda$**(dest.int.)

$\Delta l=l_2-l_1$

Δl=3.2-3=0.2m

$\Delta l=(m+1/2)\lambda$

0.2=(m+1/2)*0.1=m+1/2

m=11/2

this value do not satisfy int. conditions

because m=1,2,3,……..

prob.2:**$\Delta l=m\lambda$(cons. Int.)**

0.2=m*0.1

m= 2

this value satisfy int. conditions

the intereference is **<u>constructive.</u>**

Ex: if the distance between two slits in young s experiment(0.1mm),amonochromatic light of λ=(500nm) is used. The distance between the two slits and the screen is (1.2m),find the distance between the first bright fringe and the central fringe

Sol

D=1.2m, m=1, λ=500nm, d=0.1mm

X=?

$Xd/D=m\lambda$

$X*0.1*10^{-3}/1.2=1*500*10^{-9}$

$X=1*500*10^{-9}*1.2 /0.1*10^{-3}$ =6mm.

EX: In young s experiment if the distance between the two slits is 0.2mm,and the distance between these two slits and the screen is 1m. if the monochromatic light(λ=600nm)is incident on the slits,find the distance between two successive dark fringes?

Sol.

$\Delta x=x_{n+1} -x_n$

$\Delta x=(m+3/2)\lambda D/d - (m+1/2)\lambda D/d$

$\quad =(m+3/2-m-1/2)\lambda D/d$

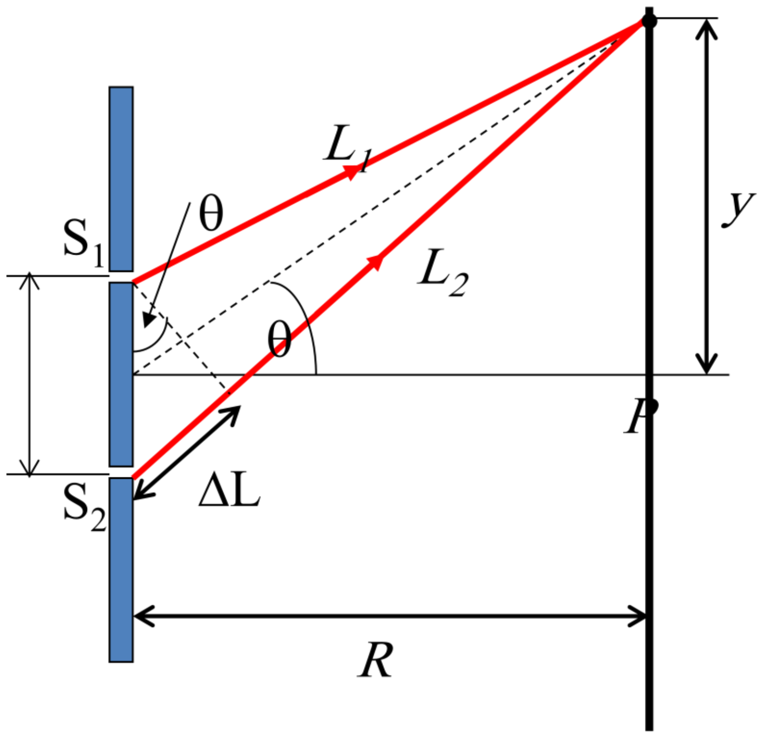
$\quad =2/2\ \lambda D/d$

$\Delta x=\lambda D/d=600*10^{-9}*1/0.2*10^{-3}$

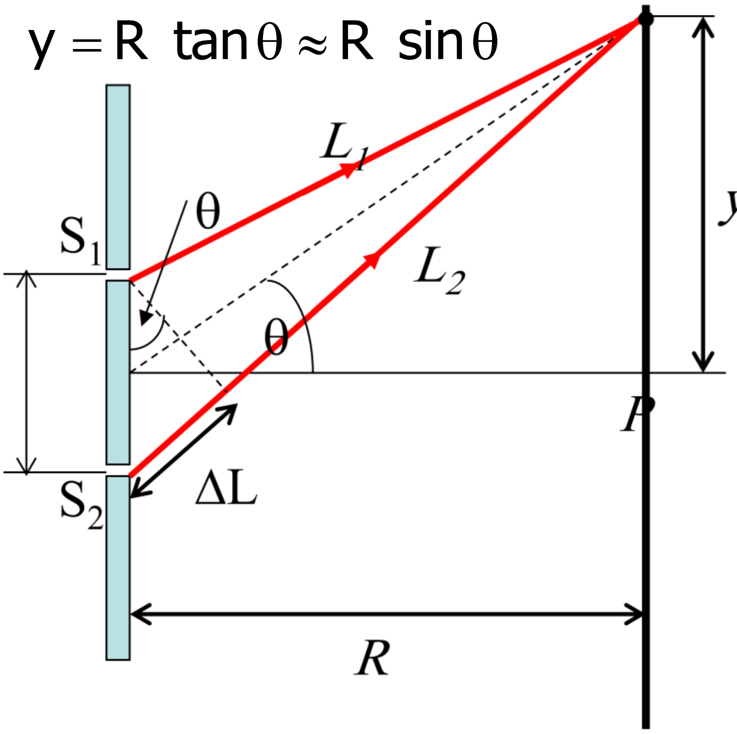
$\Delta x = 0.003m$

Example: a viewing screen is separated from the double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe (m = 2) is 4.5 cm from the center line. Find the distance between adjacent bright fringes.

Sol:



$$y = R\ \tan\theta \approx R\ \sin\theta$$



$$\tan\theta = \frac{y}{R}$$

Bright fringes:

$$m\lambda = d\ \sin\theta$$

$$m\lambda = d\ \frac{y}{R}$$

$$y = \frac{\lambda R}{d} m$$

$$y_{m+1} - y_m = \frac{\lambda R}{d}(m+1) - \frac{\lambda R}{d} m = \frac{\lambda R}{d} = \frac{(5.6\times10^{-7} m)(1.2\ m)}{(3.0\times10^{-5}\ m)} = 2.2\times10^{-2}\ m = 2.2\ cm$$

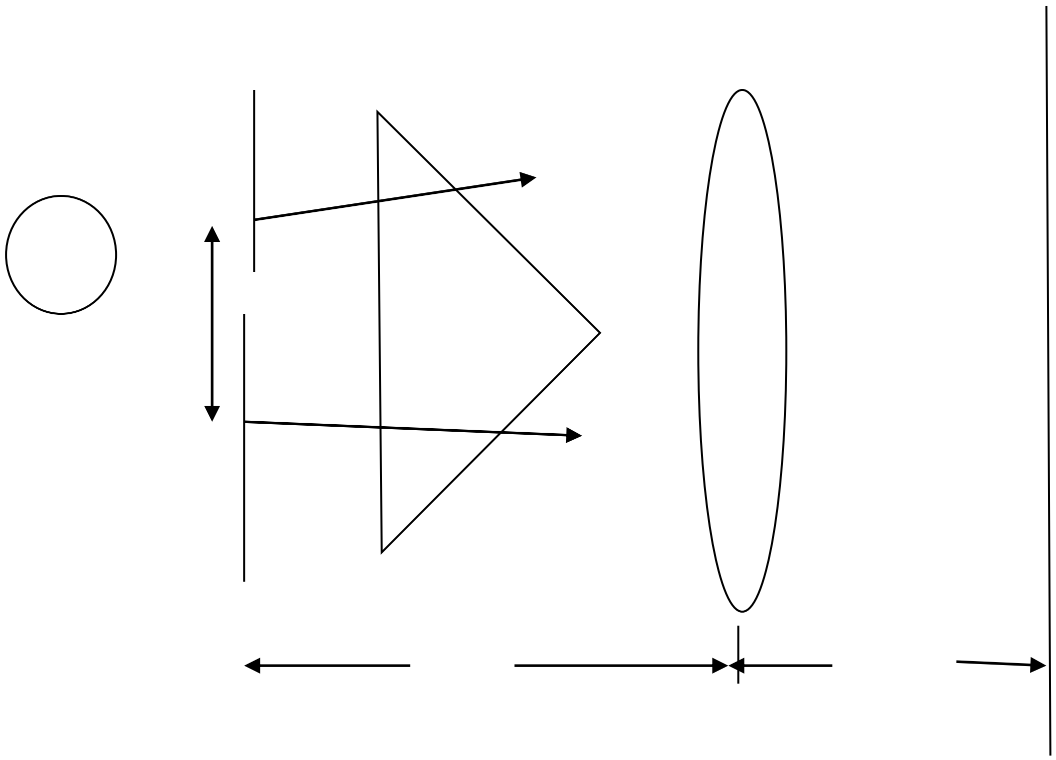
Fresnel's Biprism

The Fresnel's biprism is a prism which has one of it's angles slightly less than two right angles and two small base angles.

It acts like two very thin prisms placed base to base.

The theory for the Fresnel's Biprism is smaller to double-slit

experiment.A point source is shone through the fresnel's biprism which consists of two prisms joint at their bases to form an isosceles triangle . the resulting refraction causes the light appear as if it is coming from two point sources and similar interference pattern is created.



Other apparatus depending on division of the wave front

1-Fresnel's mirrors:-It is an arrangement in which the light from a slit is reflected in two plane mirrors slightly inclined to each other .the mirrors produce two virtual images of the slit.

2-Liyod's mirrors:-It is a mirror used to reflect part of a direct light source so that the reflection interferes with the direct source ,producing fringes.

3-split lens: other ways for dividing the wave front into two segments and subsequently recombining these at a small angles with each other.

Coherent sources

Coherent sources are those sources of light which emit continuous light waves of same wavelength,same frequency and are in same phase or have a constant phase difference.These sources produced by an oscillator.

Division of amplitude(Michelson interferometer)

interference apparatus may be conveniently divided into two main classes:

1-based on division of wave front.

2-based on division of amplitude.

The first in which the wave front is divided laterally into segments by mirrors or diaghrams.

The second in which the wave is divided by partial reflection, the Michelson interferometer is an important example of the second class.

The Michelson int. is a device that produces interference between two beams of light.A diagram of the apparatus is shown in fig(5).the basic operation of the interferometer is as follows:

Light from a light source is split into two parts.one part of the light travels adifferent path length than the other.After traversing these different path lengths,the two parts of light are brought together to interfere with each other .the interference pattern can be seen on a screen.

Light from the source strikes the beam splitter (designated by s).the beam splitter allows 50%of the radiation to be transmitted to the translatable mirror M_1.the other 50%of the radiation is reflected to the fixed mirror M_2.the compensator plate c is introduced along this path to make each path have the same optical path length when $M_1$and M_2 at the same distance from beam splitter.after returning from M_1,50% of the light is reflected toward the frosted glass screen.likewise ,50% of the light returning from M_2 is transmitted to the glass screen.at the screen,the two beams are superposed and one can observe the interference between them.

Circular fringes

To understand the fringe pattern away from the center of the screen we must distinguish between two cases:

1-when the light source is a point source and the mirrors are in exact adjustment ,we will see circular fringes.

2-if we feed the interferometer with a parallel beam then we will observe fringes of equal thickness.

Visibility of fringes

There are three principal types of measurement that can be made with the interferometer:

1-width and fine structure of spectrum lines.
2-length or displacement in terms of wavelength of light.
3-refractive indices.
For the measurements of lengh,Michelson tested the lines from various sources and concluded that a certain red line in the spectrum of cadimium was the most satisfactory.he measured the visibility,defined as:

$$V=I_{max}-I_{min} / I_{max}+I_{min}$$

Where I_{max} and I_{min} are the intensities at the maxima and minima of the fringe pattern.

Interferometric measurements of length

The principal advantage of michelson's form of interferometer over the other methods of producing interference lies in the fact that the two beams are here widely separated and the path difference can be varied at will by moving the mirror or by introducing a refracting material in one of the beams.
In Michelson int. ,when the top mirror is moved slowly from one position to another,counting the number of fringes in monochromatic light which cross the center of the field of view will give the measure of the distance the mirror has moved in terms of λ
For the position of d_1 corresponing to bright fringe of order m_1,

$$2d_1=m_1\lambda$$

And for d_2,givig a bright fringe of order m_2

$$2d_2=m_2\lambda$$

Subtracting these two eqns. We find:

$$d_1-d_2=(m_1-m_2)\lambda/2$$

hence the distance moved equals the number of fringes counted,multiplied by a half wavelength.

Twyman and Green Interferometer

It is a variant of the Michelson interferometer,principally used to test optical components.the set up of the twyman-green int. is similar to that of the Michelson int. .

There are two differences between the michelson int. and twyman int. :

1-the fixed mirror in the Michelson int. is rotatable in the twyman –green int.

2-while the light source is usually an extended source in a Michelson int. ,the light sorce is always a point source.

The rotation of one mirror results in straight fringes appearing in the interference pattern,afringing which is used to test the quality of optical components by observing changes in the fringe pattern when the component is placed in one arm of the intereferometer.

Index of refraction by interferometric methods

If a thickness t of a substance having an index of refraction n is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because of the fact that light travels more slowly in the substance and consequently has a shorter wavelength.

The optical path is now (nt) through the mediun,whereas it was practically t through the corresponding thickness of air (n=1) .

Thus the increase in optical path due to insertion of the substance is (n-1)t .this will introduce $(n-1)t/\lambda$ extra waves in the path of one beam;so if we call Δm the number of fringes by which the fringe system is displaced when the substance is placed in the beam,we have :

$$(n-1)t=(\Delta m)\lambda$$

In principle a measure of Δm,and λ thus gives a determination of n.

There are several forms of refractometes have been devised especially for this purpose,for example: Jamin, Mach-zehnder, Raleigh refractometers.

Reflection from a plane-parallel film

Let a ray of light from a source be incident on the surface of such a film at A(fig).part of this will be reflected as ray (1)and part reflected in the direction AF. upon arrival at F, part of the latter will be reflected to B and part reflected toward H. at B the ray FB will be again divided. A continuous of this process yields two sets of parallel rays, one on each side of the film. in each of these sets, of course, the intensity decreases rapidly from one ray to next. if the set of parallel reflected rays is now collected by a lens and focused at the point P, each ray will have traveled a different distance, and the phase relations may be such as to produce destructive or constructive interference at that point

It is such interference that produce the colors of thin films when they are viewed by the naked eye. in such a case L is the lens of the eye, and P lies on the retina.

The difference in optical path:-

Δ= n(AFB)- AD ....(1)

If BF is extended to intersect the perpendicular line AE at G, AF=GF because of the equality of the angles of incidence and reflection at the lower surface. thus we have:

Δ= n (GB) –AD =n (GC+CB) – AD ....(2)

Now AC is perpendicular to FB.

AC and DB represent two successive positions of a wave front reflected from the lower surface.

The optical paths must be same by any ray drawn between two wavefronts,

n (CB) =AD ....(3)

from eq.(2):

$\Delta = nGC + \cancel{nCB} - \cancel{AD}$ ....(4)

The path difference then reduces to:

$\Delta = (nGC) = n (2d\cos\varnothing^{-})$ ......(5

If this path difference is a whole number of wavelengths,we might expect rays (1)and(2) to arrive at the focus of the lens in phase with each other and produce a maxima of intensity.however we must take account of the fact that ray (1) undergoes a phase change of π at reflection,while ray(2) does not,since it is internally reflected.

The condition:

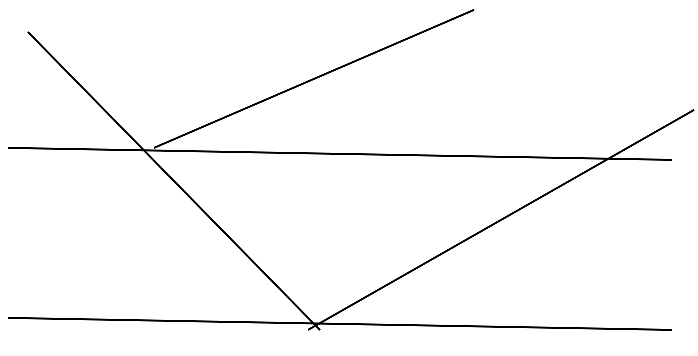
$\boxed{2nd\cos\varnothing^{-} = m\,\lambda}$ minima (destructive int.)

And:

$\boxed{2nd\cos\varnothing^{-} = (m + 1/2)\lambda}$ maxima(constructive int.)

تداخل اتلافي 2nt=	تداخل بناء 2nt=	نوع التداخل
m λ **m=1,2,3,**	**(m+1/2) λ** **m=0,1,2,**	احدى الموجتين مقلوبة
(m+1/2) λ **m=0,1,2**	**m λ** **m=1,2,3,**	الموجتان مقلوبتان او غير مقلوبتين

EX: what is the minimum value of thin film thickness (n=1.33)to for constructive interference of light(λ=521nm)

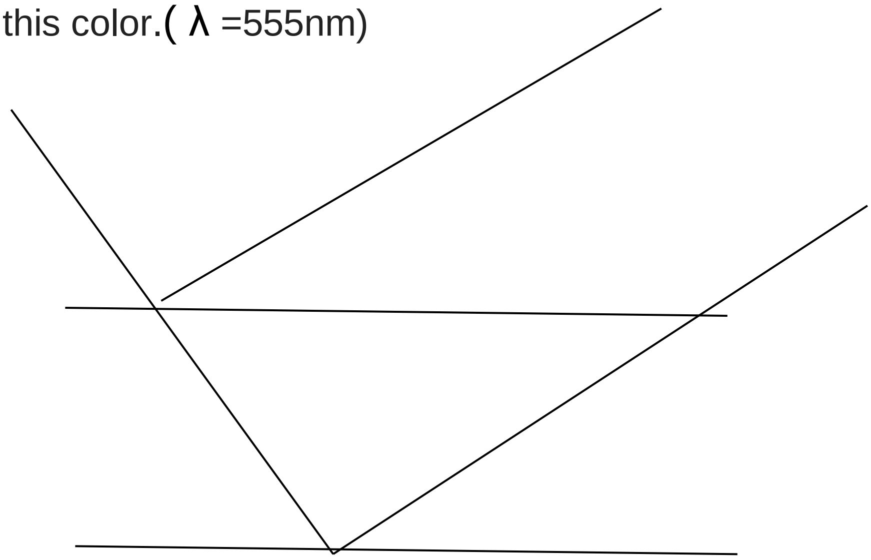


2nt=(m+1/2) λ

m=0

$t=\frac{521}{4n} = \frac{521}{4*1.333}$ =979nm

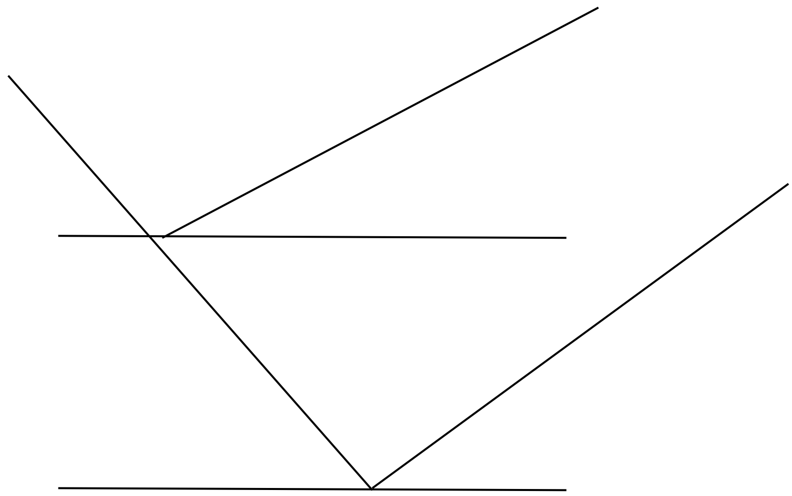
EX: Because of the presence of a layer of oil(n=1.45) on the surface of a pool of water, colored rings were observed and when looking down the pool was observed yellowish green area Calculated minimum value of thickness for the oil layer because of the appearance of this color.(λ =555nm)



2nt=(m+1/2) λ

$t= \frac{\lambda}{4n} = \frac{555nm}{4*1.45}$ =95.7

EX: Because of the presence of a layer of oil(n=1.45) on the surface of a pool of glass(n=1.55), colored rings were observed and when looking down the pool was observed yellowish green area Calculated minimum value of thickness for the oil layer because of the appearance of this color.



$2nt=m\lambda$

$t=m\lambda/2n=1*555/2*1.45$

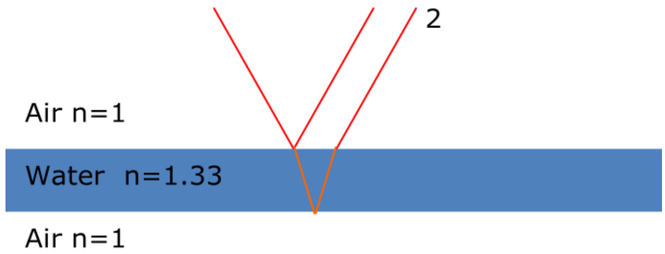
$t=192nm$

EX:

a) What is the minimum soap-film thickness (n=1.33) in air that will produce constructive interference in reflection for red (λ=652nm) light? (assume normal incidence)

b) Which visible wavelength(s) will destructively interfere when reflected from a soap film of thickness 613nm? Assume a range of 350nm to 750nm for visible light.

SOLUTION:



a- for constructive int.: $2d\cos\Theta=(m+1/2)\lambda$

$d\cos\Theta$=thickness of thin film=t

λ=wave length of light in thin film

λ_0= wavelength of light in air

$\lambda=\lambda_0/n$

$2t=(m+1/2)\lambda$

We want thinnest film possible(m=0)

2t=(1/2)*(652/1.33)

t=123nm

b-for destructive int.

$2t=m*(\lambda_0/n)$

$\lambda_0=2t\ n\ /\ m$

when m=1: λ_0=2*613*1.33/ 1 =1630 nm out of range.

When m=2:λ_0=2*613*1.33 /2=815nm out of range

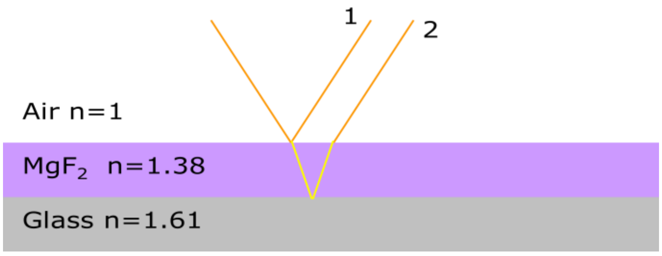
When m=3:λ_0=544nm in the range

When m=4:λ_0=407nm in the range

Ex:

A thin layer of magnesium fluoride (n=1.38) is used to coat a flint-glass lens (n=1.61).

What thickness should the MgF_2 coating be to suppress the reflection of 595nm light?



We need destructive interference (no reflection). In this case both outgoing rays reflect from a higher index, so there is no relative phase shift.

We can use any integer m>0, so start with m=0 and solve for t.

This will give the minimum thickness

$$(0+\tfrac{1}{2})\cdot\frac{595nm}{1.38}=2\cdot t\Rightarrow t_{\min}=108nm$$

To get other possible thicknesses that will work, just use

larger values for m:

Ex:

$$m=1\Rightarrow t=323nm$$
$$m=2\Rightarrow t=539nm$$
$$etc\ldots$$

Newton rings:

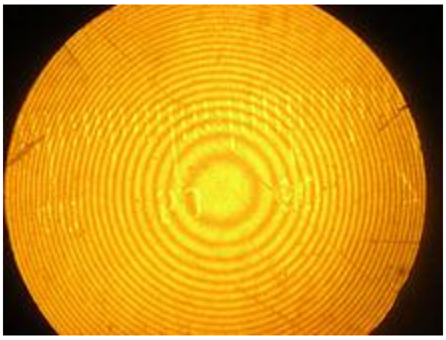
Newton's rings" is a phenomenon in which an interference pattern is created by the reflection of light between two surfaces—a spherical surface and an adjacent flat surface. It is named after Isaac Newton, who first studied them in 1717. When viewed with monochromatic light, Newton's rings appear as a series of concentric, alternating bright and dark rings centered at the point of contact between the two surfaces. When viewed with white light, it forms a concentric-ring pattern of rainbow colors, because the different wavelengths of light interfere at different thicknesses of the air layer between the surfaces.

How the interference fringes form.

The light rings are caused by constructive interference between the light rays reflected from both surfaces, while the dark rings are caused by destructive interference. Also, the outer rings are spaced more closely than the inner ones. Moving outwards from one dark ring to the next, for example, increases the path difference by the same amount, λ, corresponding to the same increase of thickness of the air layer, $\lambda/2$. Since the slope of the convex lens surface increases outwards, separation of the rings gets smaller for the outer rings. For surfaces that are not convex, the fringes will not be rings but will have other shapes.

The radius of the N^{th} Newton's bright ring is given by

$$r_N = \left[\left(N - \frac{1}{2}\right)\lambda R\right]^{1/2},$$



Ex:when we put a converging –plne lens on the plane glass sheet,we observe a special kind of circular interference fringes which is called (newton rings),follow the thickness of the variable air film between the curved surface of the lens and the glass sheet.suppose that the fringe radius much smaller than the radius of curvature of the lens surface ,and derive a formula for radius of the circular bright and dark fringes

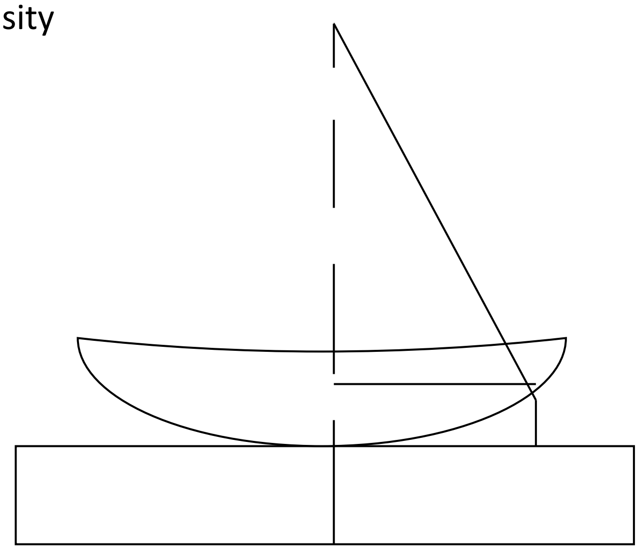
Sol:

The reflected ray from the bottom of the air film suffers a change in phase (π),because it is reflects from a medum(glass),which hase refractive index higher than the refractive index of the film(air).then the condition for the maximum intensity is:

$2d=(m+1/2)\lambda$, $m=1,2,3.....$

And the condition for minimum intensity

$2d= m\lambda, m=1,2,3,.....$



From the figure :

$d=R-(R^2-r^2)^{1/2}$

$=R-R(1-r^2/R^2)^{1/2}$

$=R-R(1-r^2/2R^2)$

$\sim r^2/2R$ ....(*)

We suppose that r<<R

$r = \sqrt{(m+1/2)\lambda r}$ constructive int.(bright fringe)

$r =\sqrt{m\lambda}$ destructive int.(dark fringe)

for destructive int.:$\Delta=2t= m\lambda$

for constructive int.:$\Delta=2t=(m+½)\lambda$

t= thickness

EX:in the newton rings experiment, the radius of curvature of the convex surface of the lens was 5.0,and it's diameter 20mm. λ=589nm

How many rings can be produce?

Sol.

To obtain a dark ring :

r= $\sqrt{m\,\lambda\,R}$

$m=\frac{r2}{\lambda R}$

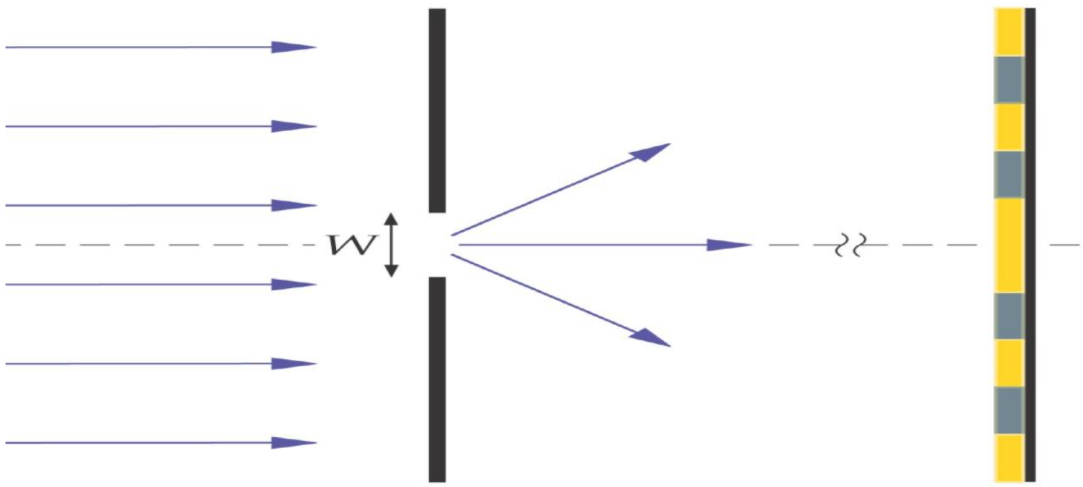
m=$\frac{(10)2}{(589*10-6)(5*103)}$

m=33.95

the number of ringes=33rings.

ch-2 / DIFFRACTION

Diffraction: is the bending of waves as they pass by an object or through an aperture.



Conditions of obtaining bright fringes and dark fringes:

d sinθ = m λ	***the condition for obtaining a dark*** fringe
dsinθ=(m+½)λ	***the condition for obtaining a bright*** fringe

where d:is the slit width

Fresnel and Fraunhofer diffraction by a single slit:

Diffraction phenomena are conveniently divided into two general classes,(1)those in which the source of light and the screen on which the pattern is observed are effectively at infinite distances from the aperture causing the diffraction, and (2)those in which either the source or the screen, or both are at finite distances from the aperture. The phenomena coming under class(1)is called(Fraunhofer diffraction, and those coming under class(2)(Fresnel diffraction). Fraunhofer diff. is much simpler to treat theoretically .

differences between Fresnels diffraction and Fraunhofer diffraction

Fresnel diff.	Fraunhofer diff.
1-thesource andthescreen are at finite distance from the diffracting aperture	1-the source and the screen are at infinite distance from the diffracting aperture
2-for obtaining fresnel diff. zone plates are used.	2- for obtaining fraunhofer l diff. plane diffractin grating are used
3-the wavefronts aredivergent either spherical or cylindrical	3-the wavefronts are plane which is released by using convex lens.
4-no mirror or lenses are usedfor observation	Diffracted light iscollected by lens
5-difficult treat theoriticall	5- much simple treat theoriticall -

Diffracting grating:

it is a device that reflects or refracts light by an amount varying to the wavelength.

it consists o f a amount of a large number of N of slits each of width a and separated from the next by a distance d.

the condition for the principal maxima:

$$d\sin\theta = m\lambda$$

m= 0, 1, 2, 3,........

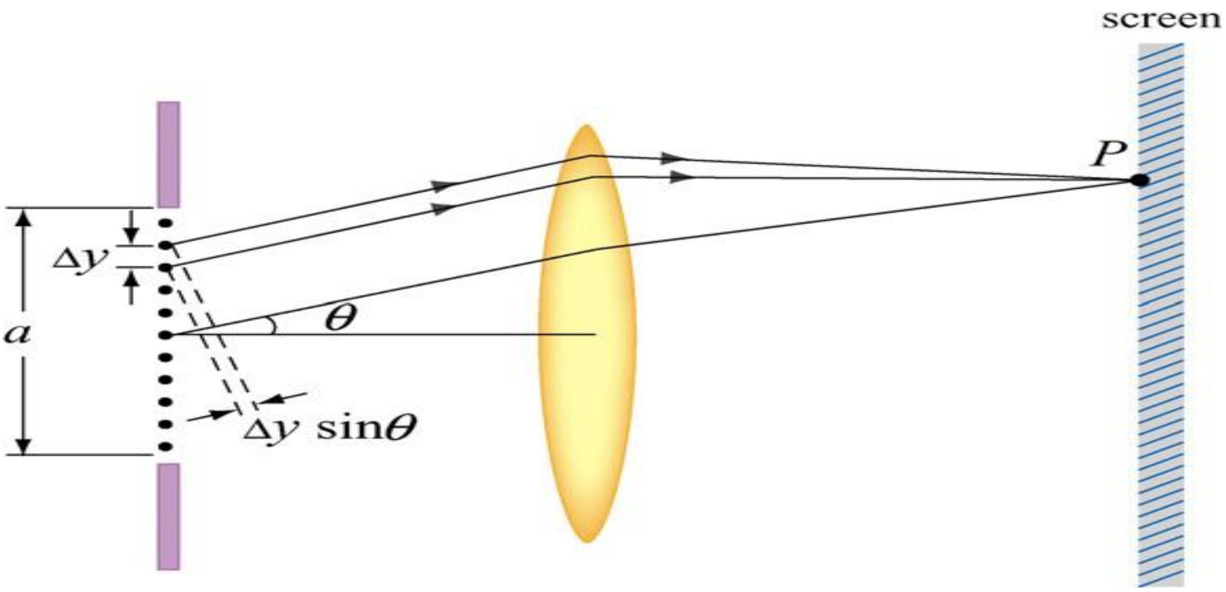
Intensity of Single-Slit Diffraction

we divide the single slit into *N* small zones, each zone have a width $\Delta y = a / N$. Two adjacent zones have a relative path difference $\delta = \Delta y \sin\theta$, and the relative phase shift $\Delta\phi$ is given by the ratio:

$$\frac{\Delta\phi}{2\pi} = \frac{\delta}{\lambda}$$

$$= \frac{2\pi\delta}{\lambda} = \frac{2\pi}{\lambda} \Delta y \sin\theta$$

$\Delta\phi$



the electric field of the first point:

$E_1 = E_0 \sin\omega t$

the electric field of the second point:

$E_2 = E_0 \sin(\omega t + \Delta\phi)$

The total electric field is:

$E = E_1 + E_2 + \dots + E_N = E_0[(\sin\omega t + \sin(\omega t + \Delta\phi) + \dots + \sin(\omega t + (N - 1)\Delta\phi]$

using some algebra and the trigonometric relation

The total electric field then becomes:

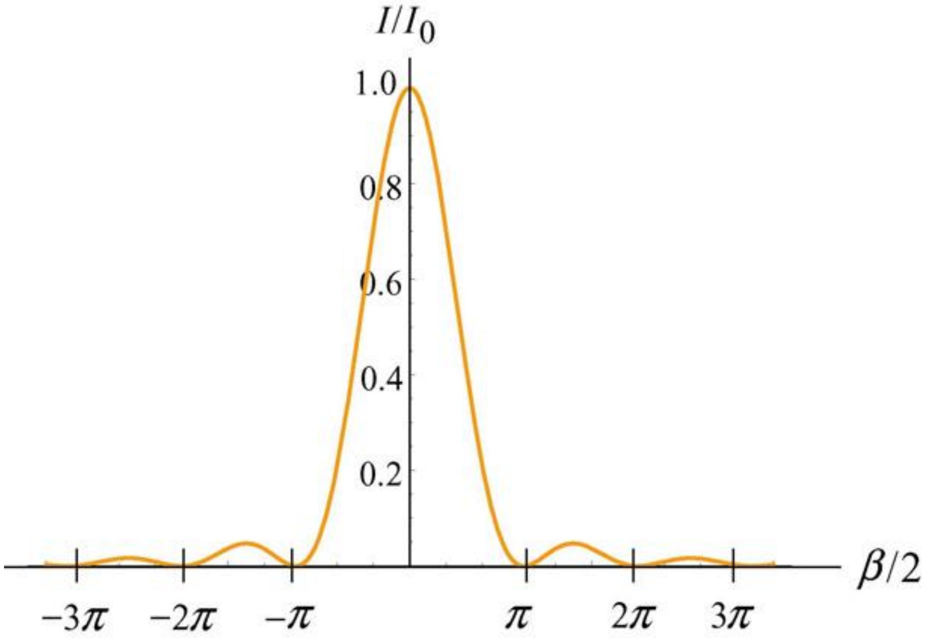
sin(ωt+(N-1

$$\mathbf{E} = E_0 \left[\frac{\sin(\frac{\phi}{2})}{\sin(\frac{\Delta\phi}{2})}\right.$$

The intensity *I* is proportional to the time average ofE^2 :

$$\mathbf{E^2} = E_0^2 \left[\frac{\sin(\frac{\phi}{2})}{\sin(\frac{\Delta\phi}{2})}\right]^2 \sin^2(\omega t+(N-1$$

and the intensity becomes



Intensity of the single-slit Fraunhofer diffraction pattern

$I=I_0\cos^2(\frac{\phi}{2})$ double

Rectangular aperture

A beam of light emerging through a small hole or aperture spreads out as it propagates. this spreading of light beam or bending of light rays is called **diffractio**n. the diffracted rays interfere constructively and destructively to form fringes. the diffraction pattern has constructive bright and dark fringes. the diffraction carries the shape of the aperture through which it is diffracting. diffraction through a circular aperture form circular fringes, diffraction through a long thin slit from straight – line fringes and diffraction through a rectangular opening form rectangular fringes.

The dimension of the slit through which light is diffracting is of the order of few millimeter (0.1 to 2mm).hence, the product of slit dimension and wavelength is very small compared to the distance between source and slit ,slit and screen. such diffraction process is known as fraunhofer diffraction.

The light intensity of the diffraction pattern is:

$$I= I\,(0)\sec^2(\alpha)\sec^2\beta$$

Where: I(0) is the intensity at the center of the pattern

α costants called interference factor

β constant called diffraction factor

the width of the central bright fringe (0^{th} order)of the diffraction pattern is given by:

$$x=\frac{2\lambda f}{a}$$

the height of the central bright fringe (0^{th}order)of the diffraction pattern is:

$$y=\frac{2\lambda f}{b}$$

where a: is the width of slit, λ: is the wavelength

b: is the height of slit, f: is the distance between slit and screen

resolving power with a rectangular aperture

resolving power: is the ability of a microscope or telescope to measure the angular separation of images that are close together .

All optical instruments give images that are affected by the diffraction at the objective lens, so if we have two points on the object that are close together it is possible that their images may possess diffraction patterns that will overlap. If they are too close the images of these two points will be indistinguishable from one another. This is especially important in astronomy where the images of two stars that are very close together need to be separated. The aperture of the telescope needs to be as large as possible to give as little diffraction as possible.

For two images of equal intensity to be resolved the central maximum (constructive interference – maximum brightness) of one diffraction pattern must fall no closer than the first minimum (destructive interference – darkness) to the centre of the second diffraction pattern (Figure 1). Using the formula for a rectangular aperture we have:

$\lambda = a \sin \Theta$ for the first minimum

where a is the aperture of the objective and λ the wavelength of radiation used

For a circular aperture and a small angle the formula has to be modified to give:

Smallest resolvable angle (φ) = $1.2\lambda/a$

The theoretical and practical resolving powers are given by

Theoretical resolving power = λ/a and

Practical resolving power = d/ D

Where λ = mean wavelength of light employed,

a = width of the rectangular slit for just resolution of two objects,

d = separation between two objects,
D = distance of the objects from the objective of the telescope hence
$\lambda/a = d / D$.

Ex: Light with a wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width 2.20x10-6m. Find the angle associated with (a) the first and (b) the second dark fringe above the central bright fringe.

Solution:

(a) First Dark Fringe, m=1

Since $a \sin\theta = m\lambda$

$2.2*10^{-6} \sin\theta=(1)\ (511*10^{-9})$

$\Theta=13.4^0$

(b) Second Dark Fringe, m=2

$a\sin\theta=m\lambda$

$2.2*10^{-6} \sin\theta= (2)(511*10^{-9})$

$\theta=27.7^0$

Ex: Monochromatic light passes through a slit of width 1.2 x 10-5 mm. If the first dark fringe of the resulting diffraction pattern is at angle $\theta=3.25^o$, what is the wavelength of the light?

Chromatic resolving power of a prism:

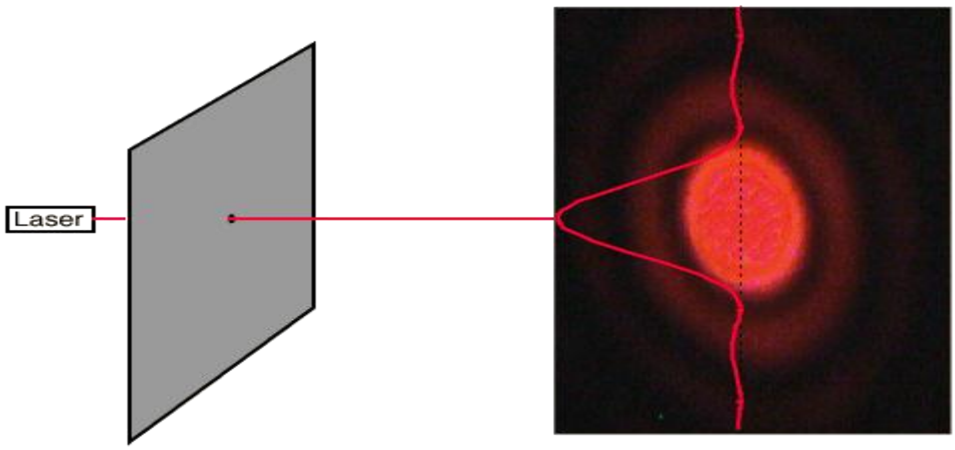
The chromatic resolving power of a prism is invariably stated for the case in which parallel rays of light are incident on the prism, in which the prism is oriented at the angle of minimum deviation at wavelength λ, and in which the complete height of the prism is utilized. The corresponding resolving power R_1 deduced on the basis of Rayleigh's criterion is:

$$R = \lambda/\Delta\lambda = bdn/\lambda$$

where n is the refractive index of the prism for the wavelength λ and b is the maximum thickness of the prism traversed by light rays. The quantities $dn/d\lambda$ and d often are called the dispersion and baselength of the prism, respectivel

circular aperture:

Circular Aperture Diffraction



When light from a point source passes through a small circular aperture, it does not produce a bright dot as an image, but rather a diffuse circular disc known as Airy's disc surrounded by much fainter concentric circular rings. This example of diffraction is of great importance because the eye and many optical instruments have circular apertures. If this smearing of the image of the point source is larger that produced by the aberrations of the system, the imaging process is said to be diffraction-limited, and that is the best that can be done with that size aperture. This limitation on the resolution of images is quantified in terms of the Rayleigh criterion so that the limiting resolution of a system can be calculated.

the intensity distribution is very much the same as that which obtained by the single-slit pattern. the dimensions of the pattern are ,however ,appreciably different from those in single-slit pattern for a slit of width equal to the diameter of the circular aperture.

For the single-slit pattern, the angular separation of the minima from the center :

$\mathrm{Sin}\theta \approx \theta = \frac{m\lambda}{b}$ where b=diameter of the aperture

Resolving power of a telescope

A telescope is an instrument used to view objects in remote areas.

The angular resolution power of a telescope is:

$$\Theta = 1.220\,\frac{\lambda}{D}$$

Where: λ- is the wavelength of light., D- is the diameter of lens aperture

Ex: calculate the minimum angle of resolution of a telescope if the diameter of the objective lens is 4cm and its focal length is 30cm,and the light wavelength is $5.6*10^{-6}$cm .

Sol.

$$\Theta = 1.22\frac{\lambda}{D} \quad = 1.22\frac{5.6*10-6}{4} \quad = 1.71*10^{-5}\text{rad}$$

Resolving power of microscope:

The resolution *R* (here measured as a distance:

$$R = \frac{1.22\lambda}{\mathrm{NA}_{\mathrm{condenser}} + \mathrm{NA}_{\mathrm{objective}}} \quad \text{where} \quad \mathrm{NA} = \eta \sin\theta$$

Here NA is the numerical aperture, θ is half the included angle of the lens, which depends on the diameter of the lens and its focal length, η is the refractive index of the medium between the lens and the specimen, and λ is the wavelength of light .

The double slit:

The **double-slit experiment,** sometimes called **Young's experiment** (after Young's interference experiment), is a demonstration that matter and energy can display characteristics of both classically defined waves and particles, and demonstrates the fundamentally probabilistic nature of quantum mechanical phenomena.

In the basic version of this experiment, a coherent light source such as a laser beam illuminates a plate pierced by two parallel slits, and the light passing through the slits is observed on a screen behind the plate. The wave nature of light causes the light waves passing through the two slits to interfere, producing bright and dark bands on the screen—a result that would not be expected if light consisted of classical particles (i.e., small chunks of matter). however, the light is always found to be absorbed at the screen at discrete points, as individual particles (not waves), the interference pattern appearing via the varying density of these particle hits on the screen. This result establishes the principle known as wave–particle duality.

equation for the intensity of double slit:

The light intensity in the double-slit interference pattern is:

$I=4A_0^2 \frac{\sin ß}{ß} \cos^2 \gamma$

The factor $\cos^2\gamma$: is the characteristic of the interference pattern produced by two beams of equal intensity.

Where $\beta=\frac{1}{2}kb\sin\Theta=(\pi b\sin\Theta)/\lambda$ and $A_0=ab/x$.the quantity β is a convenient variable, which signies one-half the phase difference between the contributions coming from opposite edges of the slit

Comparison of the single-slt and double-slit pattern

Single slit diffraction pattern	Double slit diffraction pattern
`1- The single-slit diffraction pattern is due to interference between the light passing through one half of the slit width vs. light passing through the other half.	1- The double-slit or multiple-slit interference pattern results from interference between light passing through the separated slits.
2- Intensities in single slit pattern are not constant but decreases to zero on either side of the central maxima	2- In the double slit ,the maxima appear on either side but the intensity is too weak to observe then after two or three maxima
3- Intensity pattern in single slit is quite less	3- Intensity pattern of double slit pattern is four times that of single slit pattern
4-the spacing of single slit diff. pattern depends on the width of the slit	4-spacing of double slit depend on a and b, where b is width of opacity
5-itconsistsof bright maxima and minima of gradual lower intensity	5-it consists of diffraction pattern of equally spaced interference maxima and minima within the central maxima

Distinction between interference and diffraction

INTERFERENCE	DIFFRACTION
1- Two separate wave fronts originating from two coherent sources produce interference	1- Secondary wavelets originating from different parts of the same wave front constitute diffraction
2- The region of minimum intensity is perfectly dark in interference	2- In diffraction The region of minimum intensity not perfectly dark
3- Width of the fringes is equal in interference	3- In diffraction, Width of the fringes is never equal.
4- The intensity of all positions of maxima are of the same intensity in interference.	4- In diffraction, The intensity of all positions of maxima are vary
5- in interference ,we must have two slits to produce fringes	5- in diffraction, single slit is enough to produce fringes

POLARIZATION

Introduction:

Types of waves:

We can classify the waves into two types:

1. **Longitudinal**: the thing that is waving is in the same direction as the velocity of the wave. Examples include sound waves.

2. **Transverse**: the thing that is waving is perpendicular to the velocity of the wave. Examples include water waves and waves on a string. It turns out that light is a transverse wave of electric and magnetic fields.

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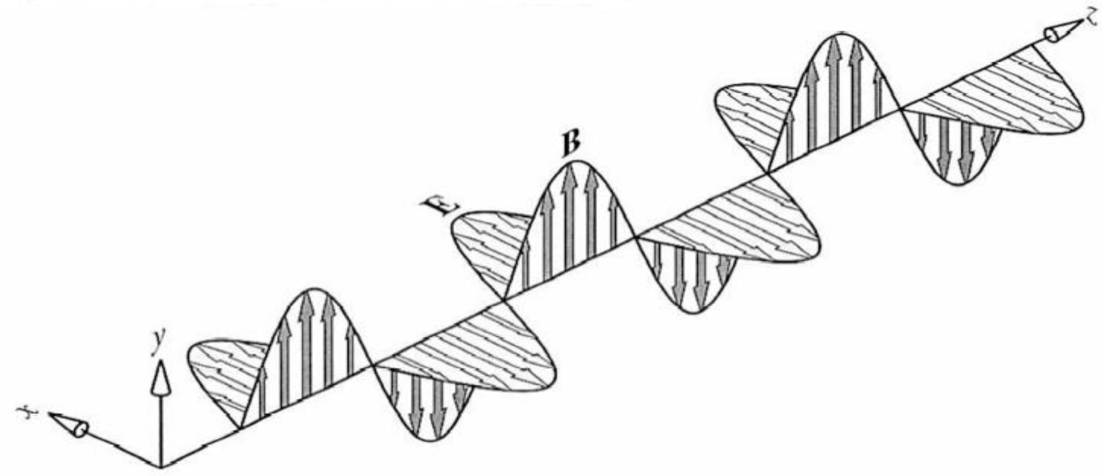


Electric & Magnetic Vector Components of Light

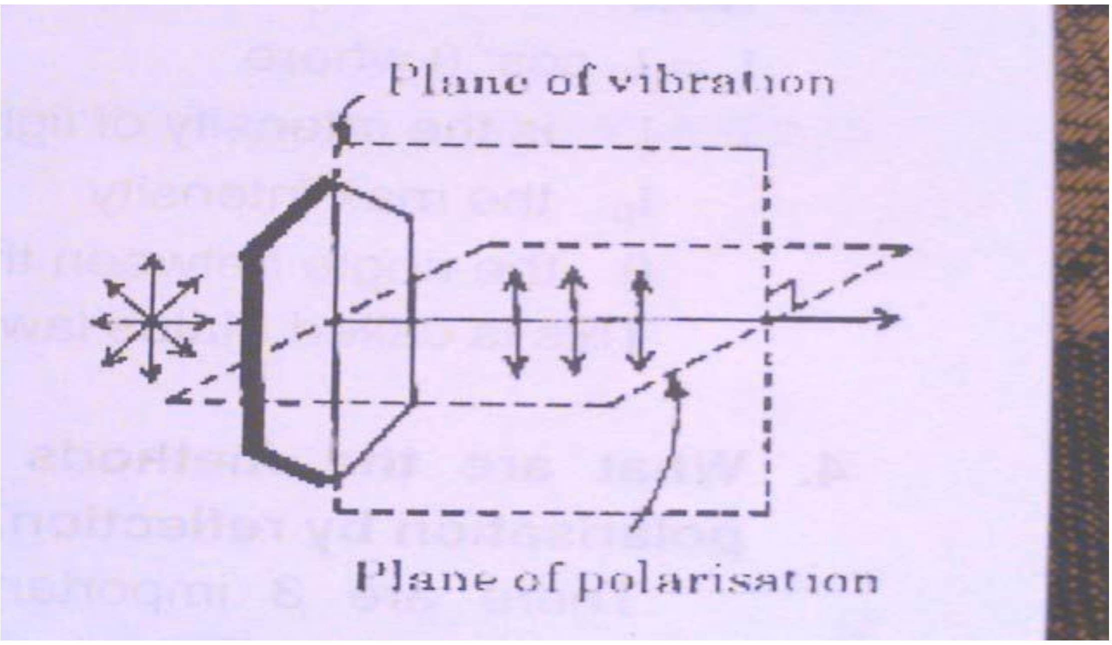
Light is a transverse electromagnetic wave motion passing in a vacuum and media. And that the source of this wave is a moving charge oscillates with a certain frequency, and with a certain direction. As a result of this oscillatory motion ,it will generate electromagnetic waves have two vectors are the electric field vector and magnetic field vector ,and the two vectors are vibrating at right angles to each other and also to the direction of light propagation , so that the cross product of the two vectors is in the direction of the wave propagation. Ordinary light or unpolarized light consists of transverse waves in which the vibrations takes place in all possible directions in a plane perpendicular to the direction of propagation of light.

Light having vibrations confined only to a single plane perpendicular to the direction of propagation of light is said to be polarized and the phenomenon is known as **polarization.**

i.e if we have an operation so that we made the vector takes a certain direction, then we say that the light has polarized. Therefore the **polarization: is a process determined by the form in which the vector will oscillates.**

The plane in which vibrations occur is called **plane of vibration**. A plane perpendicular to plane of vibration is called **plane of Polarization**. No vibrations occur in the plane of polarization**.** the ratio of intensity of polarized component of a beam to the total intensity is the degree of polarization

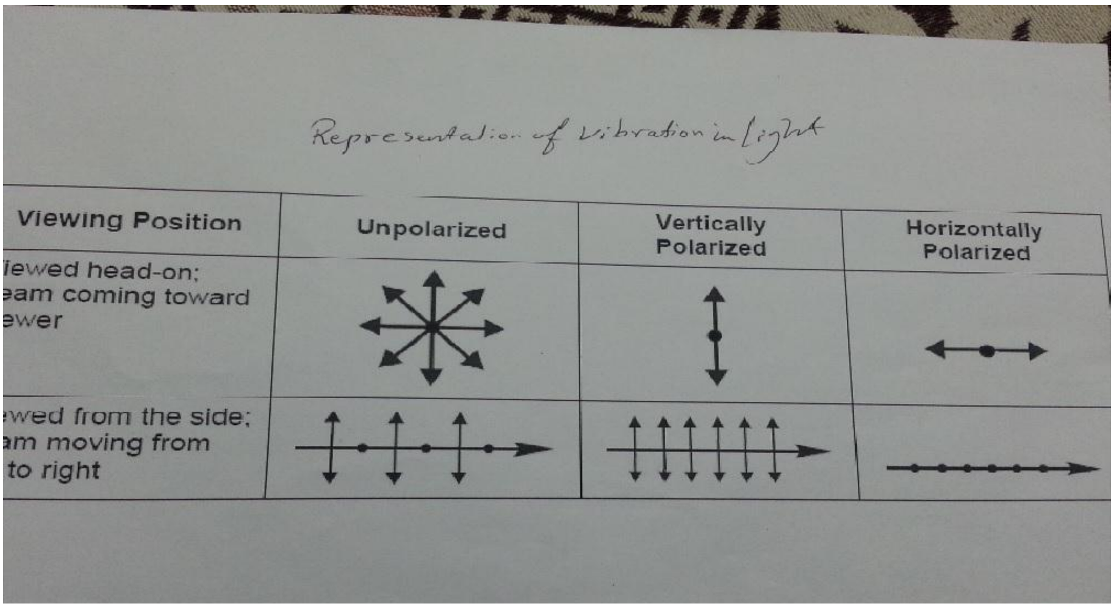
If the vector oscillation is always in a plane the is said to be(plane polarized).



Representation of the vibration in light:

According to the electromagnetic theory, any type of light consist of transverse waves, in which the oscillating magnitudes are the electric and magnetic vectors.

We can represent the vibration of light as follows:

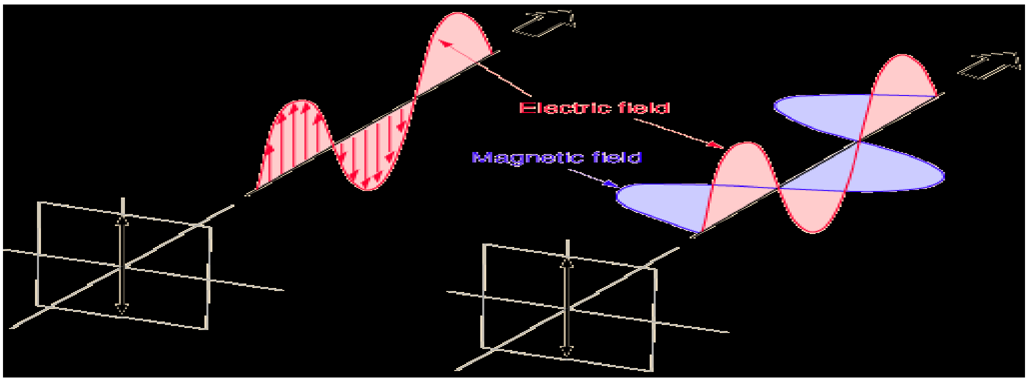


Representation of vibration in light

Viewing Position	Unpolarized	Vertically Polarized	Horizontally Polarized
iewed head-on; eam coming toward ewer			
ewed from the side; am moving from to right			

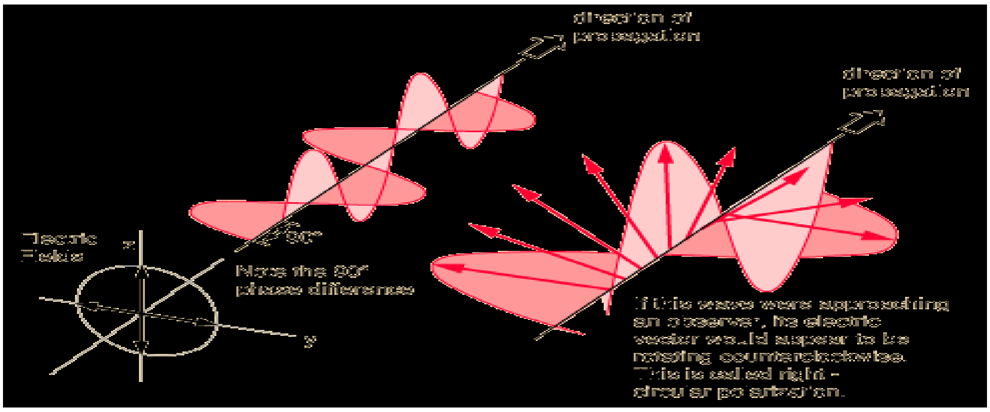
Types of polarization

1-linear polarization: For this radiation the electric field is directed along a single axis everywhere
along the light wave.
An example would be an electromagnetic wave whose fields take the form
$\mathbf{E}(z,t) = E_0\, e\, j\, (kz - t)\, \mathbf{i}$, $\mathbf{B}(z,t) = B_0\, e\, j\, (kz - t)\, \mathbf{j}$
This wave is traveling in the +z direction and is polarized along the x-axis.



• Plane EM wave – linearly polarized
• Trace of electric field vector is linear
• Also called plane-polarized light
• Convention is to refer to the electric field vector
• Weather radars usually transmit linearly polarized radiation

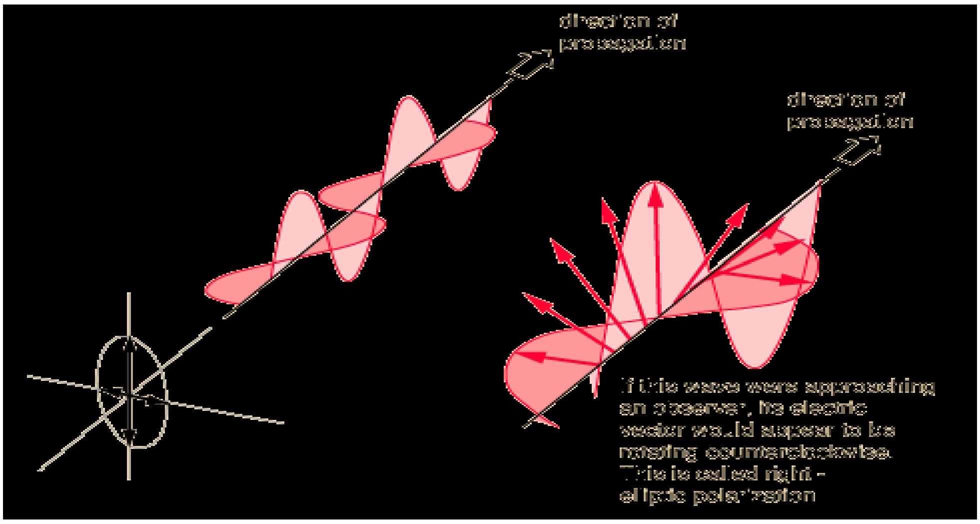
2- circular polarization



• Two perpendicular EM plane waves of equal amplitude with 90° difference in phase

• Electric vector rotates counterclockwise ➔ right-hand circular polarization

3- elliptical polarization



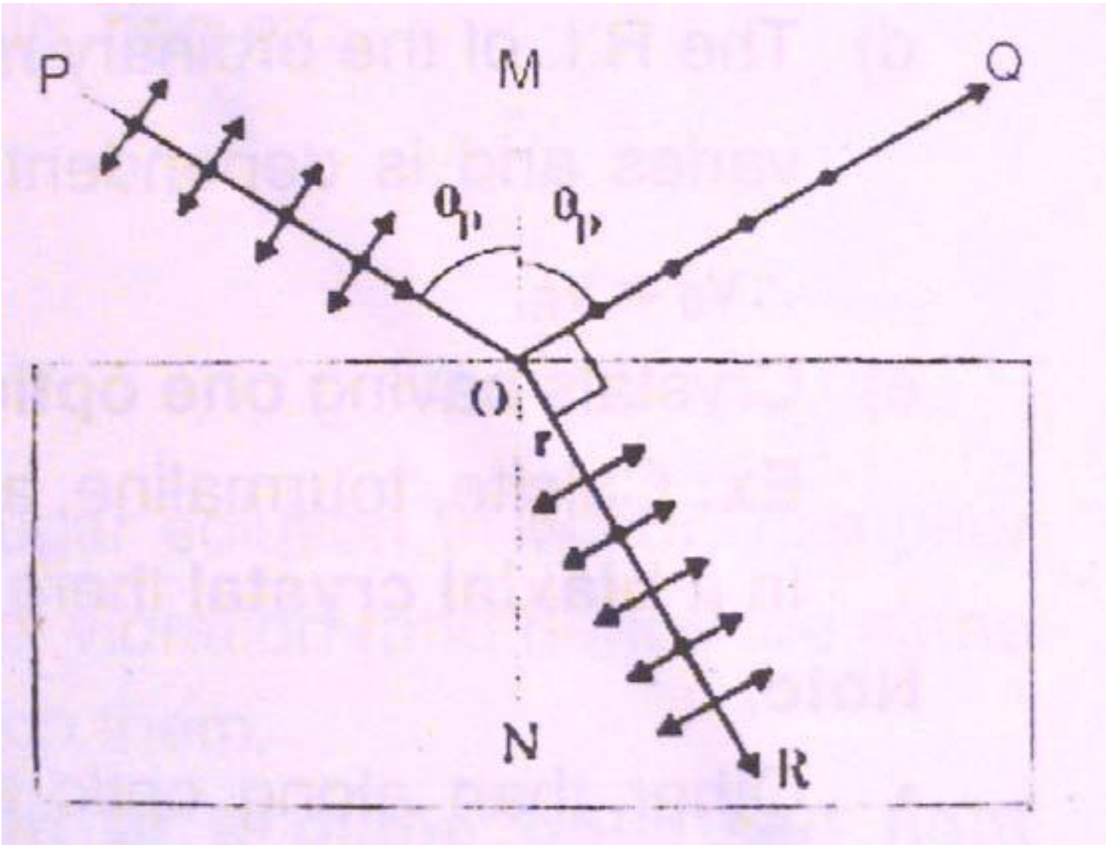
Two plane waves not in phase, either with different amplitudes and/or not 90º out of phase
• The most general state of complete polarization is elliptical

Natural EM radiation

1• Generally a mixture of different types of polarization
2• Randomly polarized component
3• Direction of the electric field vector changes randomly on very short timescale
4• ‘Unpolarized’ radiation

Polarization angle and Brewster's angle

Consider unpolarized light to be incident at an angle(Θ) on a dielectric like glass(PO),as shown in fig. below .there will always be a reflected ray (OQ) and a refracted ray OR .the reflected ray OQ is partially plane-polarized and that only at a certain definite angle. about 57^0 for ordinary glass .it was Brewster who first discovered that at this polarizing angle Θ the reflected and refracted rays are just 90^0,apart.



This remarkable discovery enables one to correlate polarization with the refractive index

where θ_1 is the angle of reflection (or incidence) and θ_2 is the angle of refraction.

Using Snell's law,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)\,,$$

one can calculate the incident angle $\theta_1 = \theta_B$ at which no light is reflected:

$$n_1 \sin(\theta_B) = n_2 \sin(90^\circ - \theta_B) = n_2 \cos(\theta_B)\,.$$

Solving for θ_B gives

$$\frac{\sin\theta}{\cos\theta} = n$$

$$\boxed{\tan\theta = n} \quad \text{Brewster's law}$$

this is Brewster's law: ***The refractive index of the medium is equal to the tangent of the polarizing angle*** ,which shows that the angle of incidence for maximum polarization depends only on the refractive index.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$\theta_1 + \theta_2 = 90^\circ$ at $\Rightarrow \theta_1 = \theta_p$

$$\theta_2 = 90^\circ - \theta_1$$

عند زاوية بروستر فأن الجزء المنكسر من الشعاع الساقط يكون عموديا على الجزء المنعكس اي ان الزاوية بين الجزء المنعكس والمنكسر $= 90^\circ$

$$n_1 \sin\theta_B = n_2 \sin(90 - \theta_B)$$

$$n_1 \sin\theta_1 = n_2 \cos\theta_1$$

$$n = \frac{n_2}{n_1} = \frac{\sin\theta_B}{\cos\theta_B} = \tan\theta_B$$

$n_1 = 1$ هواء

$$\boxed{n = \tan\theta_B}$$

EX:-

unpolarized light is to be reflected off a glass surface ($n = 1.5$), Determine the Brewster angle for each instance

Sol:-

a- light going from air to glass:-

$n_1 = 1 \quad n_2 = 1.5$

$$\tan\theta_B = \frac{n_2}{n_1} = \frac{1.5}{1} = 1.5$$

$$\theta_B = \tan^{-1} 1.5 = 56.3$$

b- light going from glass to air

$n_1 = 1.5, \ n_2 = 1$

$$\tan\theta_B = \frac{n_2}{n_1} = \frac{1}{1.5} = 0.667$$

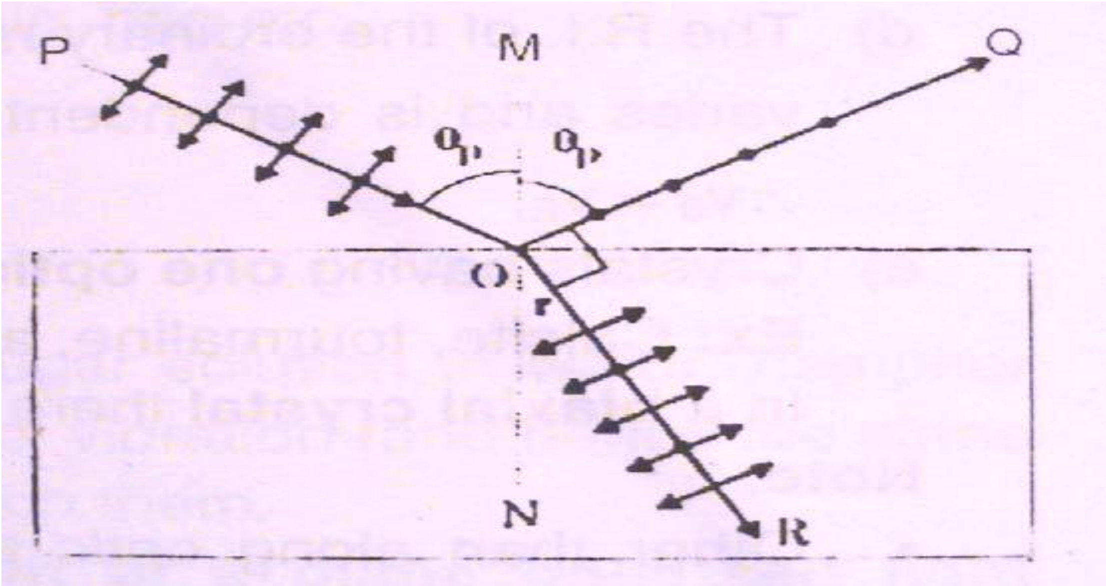
$$\theta_B = \tan^{-1}(0.667) = 33.7^\circ$$

Methods of producing plane polarized light

The different methods of producing plane polarized light are
i. Reflection
ii. Refraction
iii. Double refraction and
iv. Selective absorption

1. Polarization by Reflection:-

When ordinary or unpolarized light is made to fall on the surface, (transparent) part of the light is reflected and the other part is refracted. For small angles of incidence the reflected ray is partially polarized. The degree of polarization depends on the angle of incidence. As angle of incidence is increased, the degree of polarization also increases. For one particular angle of incidence called the polarizing angle $P\theta$, the reflected light is completely plane polarized. This was discovered by Malus in the Year 1808. Later Brewster showed that at polarizing angle $P\theta$, the reflected and refracted light are perpendicular to each other. Polarized reflected light has vibrations parallel to surface and perpendicular to the plane of paper. The refracted ray is partially polarized.

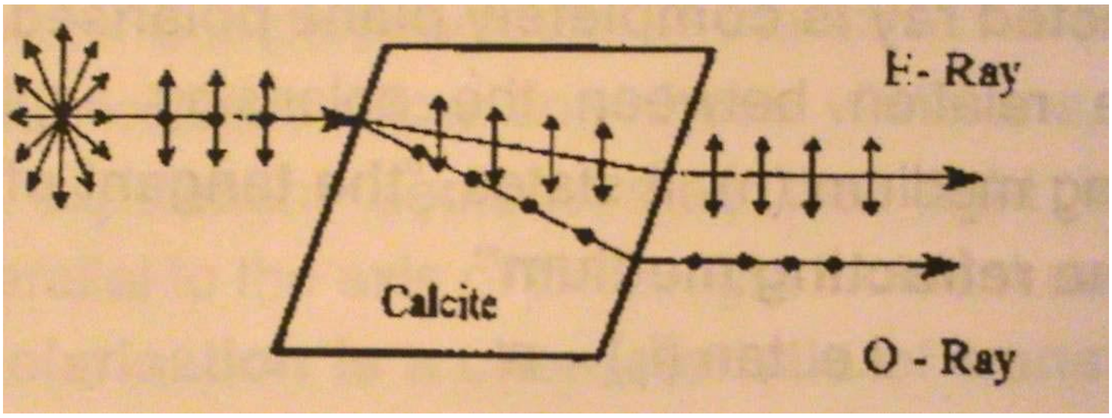


2. Polarization by pile of plates:-

Plane polarized light can be produced using pile of plates arrangement. The pile of plates arrangement consists of number of thin glass plates placed parallel to one another obliquely inside a hallow tube. When a beam of unpolarized light is incident on the 1st plate at polarizing angle, the 1st glass plate reflects 15% of the incident vibrations lying in a plane perpendicular to the plane of incidence and transmits 85% of them 100% of the vibrations in the plane of incidence are transmitted. This process is repeated at each glass plate. As a result the light emerging out of the last plate is nearly 100% plane polarized

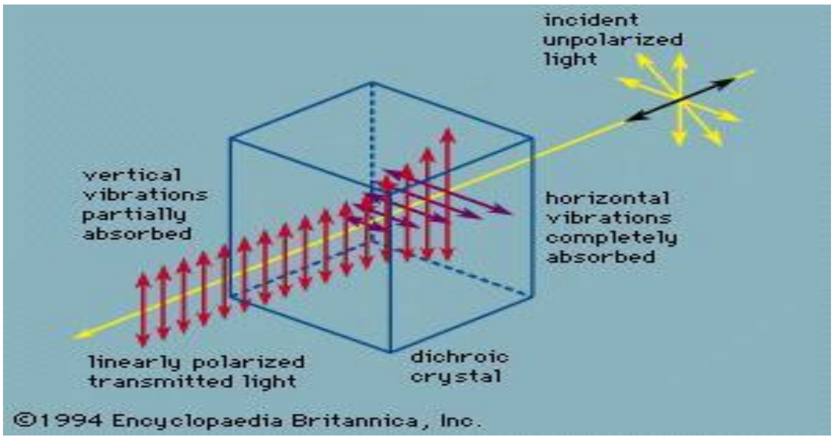
3- Polarization by Double refraction:-

Certain crystals like quartz, mica, calcite have the property of producing two refracted rays for every ray that is incident on them.
The phenomenon of refraction where in two refracted rays are produced for a given incident ray is called double refraction or birefringence. The two refracted rays are plane polarized in mutual perpendicular planes. By eliminating one of the them, plane polarized light can be obtained. The crystals which exhibit this property are called doubly refracting crystals.



4- Polarization by Selective absorption (or) Dichroism:-

When unpolarized light is incident on a doubly refracting crystal it splits up into ordinary and extra-ordinary ray that are plane polarized in mutually perpendicular planes. As these rays pass through a crystal of suitable thickness, they are absorbed to different extents. This property is called as dichroism. And the crystal is said to be dichroic.
E. g. Tourmaline crystal



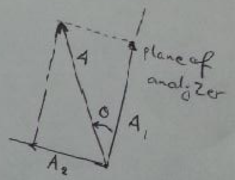
law of MALUS

(P - 05)

Law of MALUS

This law tells us how the intensity transmitted by the analyzer varies with the angle that it's plane of transmission makes with that of the polarizer.

Proof :-



let A represent the amplitude transmitted by the polarizer for which the plane of the ~~figure~~ transmission intersects the plane of the figure in the vertical dashed line

when this light strikes the analyzer ; set at the angle θ , one can resolve the incident amplitude into two components A_1, and A_2 .

A_2 is eliminated in the analyzer.

∴ The amplitude of the light ~~than~~ passes through the analyzer is therefore :- $A_1 = A\cos\theta$

$$I = A_1^2 = A^2\cos^2\theta = I_0\cos^2\theta$$

I_0 the intensity of the incident polarized light

I := intensity of light that pass through the polarizer

$$\boxed{I = I_0\cos^2\theta}$$ Malus law

θ:- angle between the transmission axis of the polarizer and the direction of the E-field vibration

* For crossed polarizers, $\theta = 90°$, so no light is transmitted

$$I = I_o \cos^2(90) = I_o(0) = 0$$